Name: Key

Date: 10/15

Direct Variation
Common Core Algebra II

We begin our linear unit by looking at the simplest linear relationship that can exist between two variables, namely that of **direct variation**. We say that two variables are **directly related** or **proportional** to one another if the following relationship holds.

Proportional or Direct Relationships

Two variables, x and y, have a **direct (proportional) relationship** if for every ordered pair (x, y) we have:

$$\boxed{\frac{y}{x}=k \quad \text{or} \quad y=kx}$$

Stated succinctly, y will always be a constant multiple of x. The value of k is known as the **constant of variation**.

***Exercise* #1:** In each of the following, x and y are directly related. Solve for the missing value.

(a) $y=15$ when $x=5$

$y=?$ when $x=9$

$\frac{15}{5}=\frac{y}{9}$

$y=27$

(b) $y=-6$ when $x=4$

$y=?$ when $x=-10$

$\frac{-6}{4}=\frac{y}{-10}$

$y=15$

(c) $y=12$ when $x=16$

$y=?$ when $x=24$

$\frac{12}{16}=\frac{y}{24}$ $y=18$

***Exercise* #2:** The distance a person can travel varies directly with the time they have been traveling if going at a constant speed. If Phoenix traveled 78 miles in 1.5 hours while going at a constant speed, how far will he travel in 2 hours at the same speed?

$\frac{78}{1.5}=\frac{d}{2}$ $d=104$ miles

***Exercise* #3:** Jenna works a job where her pay varies directly with the number of hours she has worked. In one week, she worked 35 hours and made \$274.75. How many hours would she need to work in order to earn \$337.55?

$\frac{35}{274.75}=\frac{h}{337.55}$ $h=43$ hours

We will now examine the graph of a direct relationship and see why it is indeed the simplest of all linear functions.

***Exercise* #4:** Two variables, x and y, vary directly. When $x = 6$ then $y = 4$. The point is shown plotted below.

(a) Find the y-values for each of the following x-values. Plot each point and connect.

$x = 3$

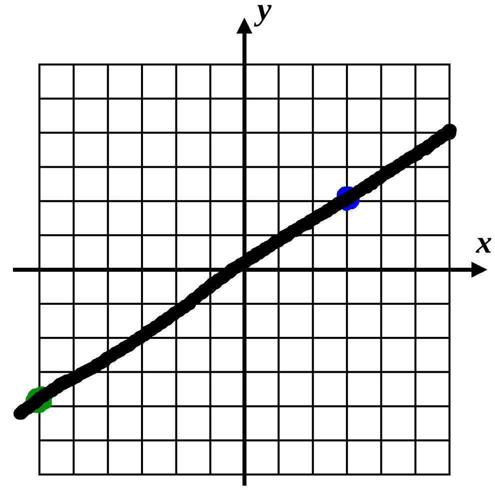
$\frac{4}{6} = \frac{y}{3}$

$y = 2$

$x = -6$

$\frac{4}{6} = \frac{y}{-6}$

$y = -4$



(b) What is the constant of variation in this problem? What does it represent on this line?

$k = \frac{y}{x} = \frac{4}{6} = \frac{2}{3}$ Slope

(c) Write the equation of the line you plotted in (a).

$y = mx + b$ $y = \frac{2}{3}x$

Direct relationships often exist between two variables whose values are zero simultaneously.

***Exercise* #3:** The miles driven by a car, d, varies directly with the number of gallons, g, of gasoline used. Abagail is able to drive $d = 336$ miles on $g = 8$ gallons of gasoline in her hybrid vehicle.

(a) Calculate the constant of variation for the relationship $\frac{d}{g}$. Include proper units in your answer.

$\frac{336}{8} = 42$ mpg

write an equation

(b) Give a linear equation that represents the relationship between d and g. Express your answer as an equation solved for d.

$d =$

$d = 42g$

(c) How far can Abagail drive on $g = 6$ gallons of gas?

$d = 42(6)$

$d = 252$ miles

(d) How many gallons of gas will Abagail need in order to drive 483 miles?

$\frac{483}{42} = \frac{42g}{42}$

$g = 11.5$ gallons

Name: ____________________________________ Date: __________________

DIRECT VARIATION
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. In each of the following, the variable pair given **are proportional** to one another. Find the missing value.

(a) $b = 8$ when $a = 16$
$b = ?$ when $a = 18$

(b) $y = 10$ when $x = 14$
$y = ?$ when $x = 21$

(c) $w = -2$ when $u = 6$
$w = ?$ when $u = -15$

2. In the following exercises, the two variables given **vary directly** with one another. Solve for the missing value.

(a) $p = 12$ when $q = 8$
$p = ?$ when $q = 6$

(b) $y = 21$ when $x = 9$
$y = ?$ when $x = -6$

(c) $z = -5$ when $w = 2$
$z = ?$ when $w = 8$

3. If *x* and *y* vary directly and $y = 16$ when $x = 12$, then which of the following equations correctly represents the relationship between *x* and *y*?

(1) $y = \frac{3}{4}x$ (3) $xy = 192$

(2) $y + x = 28$ (4) $y = \frac{4}{3}x$

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APPLICATIONS

4. The distance Max's bike moves is directly proportional to how many rotations his bike's crank shaft has made. If Max's bike moves 25 feet after two rotations, how many feet will the bike move after 15 rotations?

5. For his workout, the increase in Jacob's heart rate is directly proportional to the amount of time he has spent working out. If his heartbeat has increased by 8 beats per minute after 20 minutes of working out, how much will his heartbeat have increased after 30 minutes of working out?

6. When a photograph is enlarged or shrunken, its width and length stay proportional to the original width and length. Rojas is enlarging a picture whose original width was 3 inches and whose original length was 5 inches. If its new length is to be 8 inches, what is the exact value of its new width in inches?

7. For a set amount of time, the distance Kirk can run is directly related to his average speed. If Kirk can run 3 miles in while running at 6 miles per hour, how far can he run in the same amount of time if his speed increases to 10 miles per hour?

REASONING

8. Two variables are proportional if they can be written at $y = kx$, where k is some constant. This leads to the fact that when $x = 0$ then $y = 0$ as well. Is the temperature measured in Celsius proportional to the temperature measured in Fahrenheit? Explain.